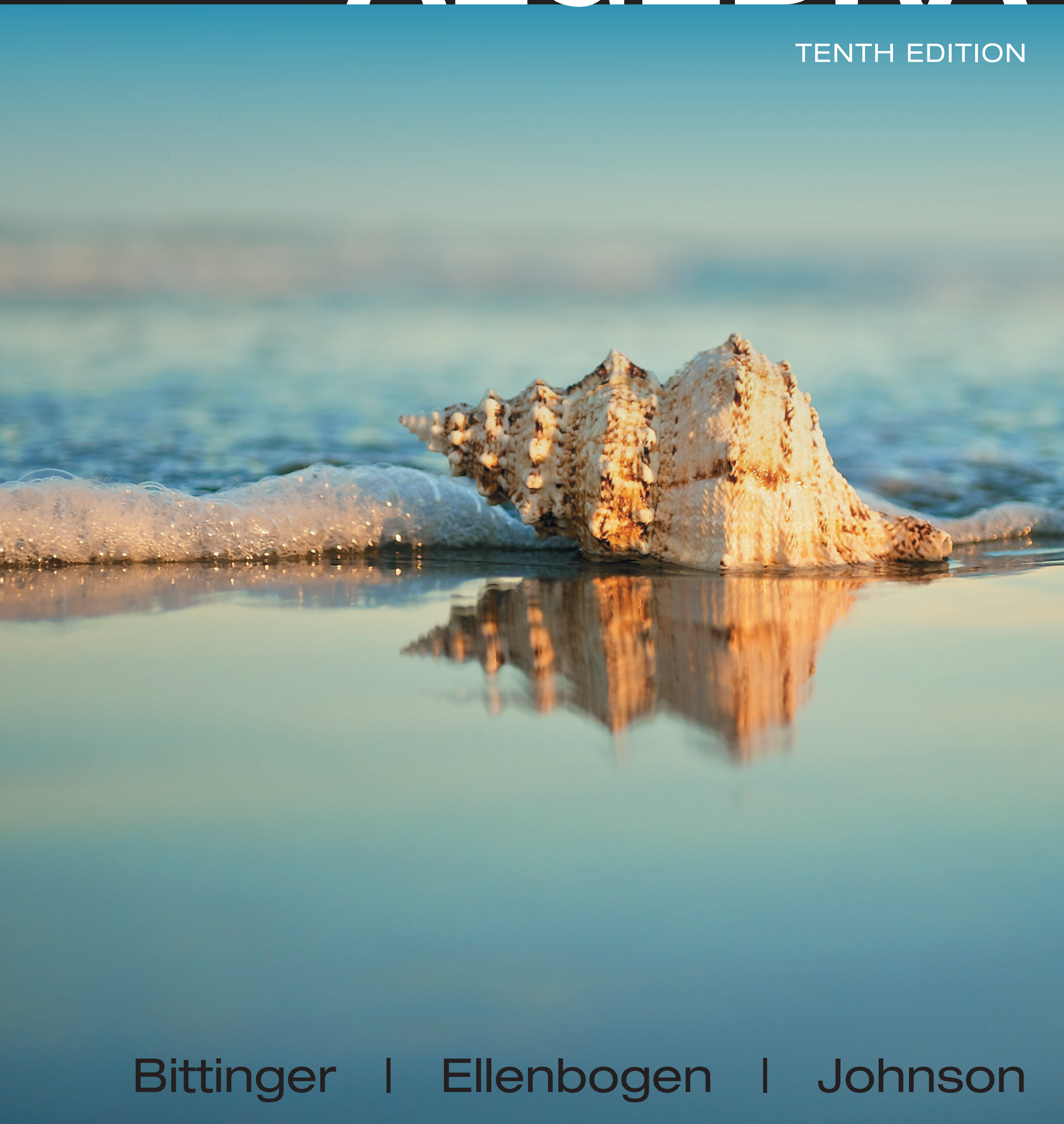
Intermediate ALGEBRA

TENTH EDITION

CONCEPTS AND APPLICATIONS



Bittinger | Ellenbogen | Johnson

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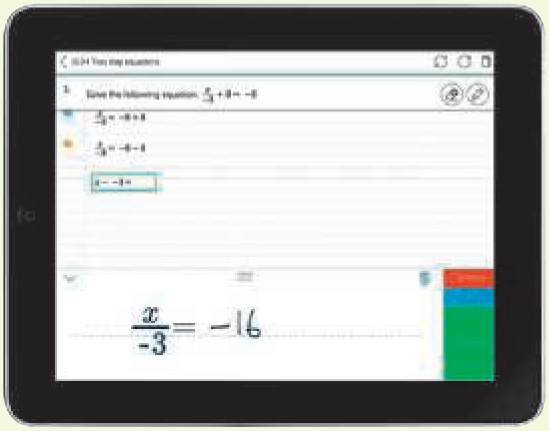
Resources for Success

MyMathLab® Online Course

The course for *Intermediate Algebra: Concepts and Applications,* 10th Edition, includes all of MyMathLab's robust features and functionality, plus these additional highlights.

New! Workspace

Workspace Assignments allow students to work through an exercise step by step, showing their mathematical reasoning. Students receive immediate feedback after they complete each step, and helpful hints and videos are available for guidance, as needed. When students access Workspace using a mobile device, handwriting-recognition software allows them to write out answers naturally using their fingertip or a stylus.



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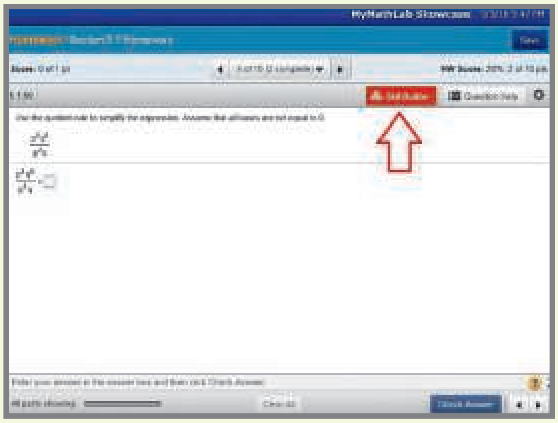
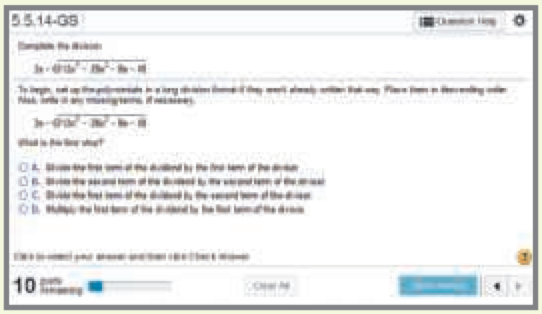
New! Skill Builder Adaptive Practice

When a student struggles with assigned homework, Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. When the system has determined that the student has a high probability of successfully completing the assigned exercise, it suggests that the student return to the assignment. When Skill Builder is enabled for an assignment, students can choose to do the extra practice without being prompted. This new feature allows instructors to assign fewer questions for homework so that students can complete as many or as few questions as needed.





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Intermediate Algebra

Concepts and Applications

Tenth Edition

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Editorial and Production Services:	Martha Morong/Quadrata, Inc.
Composition:	Cenveo® Publisher Services
Illustrations:	Network Graphics
Cover Design:	Cenveo® Publisher Services
Cover Image:	© Getty Images/Robert D. Barnes

Library of Congress Cataloging-in-Publication Data

Bittinger, Marvin L. | Ellenbogen, David. | Johnson, Barbara L.
Intermediate algebra : concepts & applications / Marvin L. Bittinger,
Indiana University Purdue University Indianapolis, David J. Ellenbogen,
Community College of Vermont, Barbara L. Johnson,
Ivy Tech Community College of Indiana
Intermediate algebra
10th edition. | Boston : Pearson, c2018.
LCCN 2016020677| ISBN 9780134497174 (hardcover: student edition) | ISBN 0134497171
(hardcover: student edition) | ISBN 978013450737-8 (hardcover: AIE) |
ISBN 0134450737-1 (hardcover: AIE)
LCSH: Algebra—Textbooks.
LCC QA154.3 .B57 2018 | DDC 512.9—dc23
LC record available at https://lccn.loc.gov/2016020677

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Pearson

ISBN 13: 978-0-13-449717-4
ISBN 10: 0-13-449717-1

Contents

A KEY TO THE ICONS IN THE EXERCISE SETS

Concept reinforcement exercises, indicated by blue exercise numbers, provide basic practice with the new concepts and vocabulary.

Aha! Exercises labeled Aha! indicate the first time that a new insight can greatly simplify a problem and help students be alert to using that insight on following exercises. They are not more difficult.

Calculator exercises are designed to be worked using either a scientific calculator or a graphing calculator.

Graphing calculator exercises are designed to be worked using a graphing calculator and often provide practice for concepts discussed in the Technology Connections.

Writing exercises are designed to be answered using one or more complete sentences.

✓ A check mark in the annotated instructor's edition indicates Synthesis exercises that the authors consider particularly beneficial for students.

The research icon indicates an exercise in which students are asked to use research skills to extend or to explore further applications from the text.

Preface

Welcome to the tenth edition of *Intermediate Algebra: Concepts and Applications*, one of three programs in an algebra series that also includes *Elementary and Intermediate Algebra: Concepts and Applications,* Seventh Edition, and *Elementary Algebra: Concepts and Applications,* Tenth Edition. As always, our goal is to present the content of the course clearly yet with enough depth to allow success in future courses. You will recognize many proven features, applications, and explanations; you will also find new material developed as a result of our experience in the classroom as well as from insights from faculty and students.

Understanding and Applying Concepts

Our goal is to help today's students learn and retain mathematical concepts. To achieve this, we feel that we must prepare students in developmental mathematics for the transition from "skills-oriented" elementary algebra courses to more "concept-oriented" college-level mathematics courses. This requires the development of critical thinking skills: to reason mathematically, to communicate mathematically, and to identify and solve mathematical problems.

Following are aspects of our approach that we use to help meet the challenges we all face when teaching developmental mathematics.

Problem Solving We use problem solving and applications to motivate the students wherever possible, and we include real-life applications and problem-solving techniques throughout the text. Problem solving encourages students to think about how mathematics can be used, and it helps to prepare them for more advanced material in future courses.

In Chapter 1, we introduce our five-step process for solving problems: (1) Familiarize, (2) Translate, (3) Carry out, (4) Check, and (5) State the answer. Repeated use of this problem-solving strategy throughout the text provides students with a starting point for any type of problem they encounter, and frees them to focus on the unique aspects of the particular problem. We often use estimation and carefully checked guesses to help with the *Familiarize* and *Check* steps (see pp. 169 and 394).

Applications Interesting, contemporary applications of mathematics, many of which make use of real data, help motivate students and instructors. In this new edition, we have updated real-world data examples and exercises to include subjects such as website design (p. 123), college readiness (p. 195), and bald eagles (p. 636). For a complete list of applications and the page numbers on which they can be found, please refer to the Index of Applications at the back of the book.

Conceptual Understanding Growth in mathematical ability includes not only mastering skills and procedures but also deepening understanding of mathematical concepts. We are careful to explain the reasoning and the principles behind procedures and to use accurate mathematical terminology in our discussion. In addition, we provide a variety of opportunities for students to develop their understanding of mathematical concepts, including making connections between concepts, learning through active exploration, applying and extending concepts, using new vocabulary, communicating comprehension through writing, and employing research skills to extend their examination of a topic.

Guided Learning Path

To enhance the learning process and improve learner outcomes, our program provides a broad range of support for students and instructors. Each person can personalize his or her learning or teaching experience by accessing help when he or she needs it.

PREPARE: Studying the Concepts

Students can learn about each math concept by reading the textbook or etext, watching the To-the-Point Objective videos, participating in class, working in the *MyMathGuide* workbook—or using whatever combination of these course resources works best for him or her.

Enhanced! **Text** The exposition, examples, and exercises have been carefully reviewed and, as appropriate, revised or replaced. New features (see below) include more systematic review and preparation for practice, as well as stronger focus on the real-world applications for the math.

Enhanced! **MyMathLab** has been greatly expanded for this course, including adding more ways for students to personalize their learning path so they can effectively study, master, and retain the math. (See pp. xiv–xv for more details.)

To-the-Point Objective Videos is a comprehensive program of objective-based, interactive videos that can be used hand-in-hand with the *MyMathGuide* workbook. Video support for Interactive Your Turn exercises in the videos prompts students to solve problems and receive instant feedback.

MyMathGuide: Notes, Practice, and Video Path is an objective-based workbook (available in print and in MyMathLab) for guided, hands-on learning. It offers vocabulary, skill, and concept review; and problem-solving practice with space for students to fill in the answers and stepped-out solutions to problems, show their work, and write notes. Students can use *MyMathGuide*—while watching the videos, listening to the instructor's lecture, or reading the textbook or etext—to reinforce and self-assess their learning.

PARTICIPATE: Making Connections through Active Exploration

Knowing that developing a solid grasp of the big picture is a key to student success, we offer many opportunities for active learning to help students practice, review, and confirm their understanding of key concepts and skills.

New! **Chapter Opener Applications with Infographics** use current data and applications to present the math in context. Each application is related to exercises in the text to help students model, visualize, learn, *and* retain the math. We also added many new spotlights on real people sharing how they use math in their careers.

Algebraic–Graphical Connections, which appear occasionally throughout the text, draw explicit connections between the algebra and the corresponding graphical visualizations. (See pp. 154 and 504.)

Exploring the Concept, appearing once in nearly every chapter, encourages students to think about or visualize a key mathematical concept. (See pp. 171 and 480.) These activities lead into the **Active Learning Figure** interactive animations available in MyMathLab. Students can manipulate Active Learning Figures through guided and open-ended exploration to further solidify their understanding of these concepts.

Connecting the Concepts summarizes concepts from several sections or chapters and illustrates connections between them. Appearing at least once in every chapter, this feature includes a set of mixed exercises to help students make these connections. (See pp. 261 and 339.)

Technology Connection is an optional feature in each chapter that helps students use a graphing calculator or a graphing calculator app to visualize concepts. Exercises are included with many of these features, and additional exercises in many exercise sets are marked with a graphing calculator icon to indicate more practice with this optional use of technology. (See pp. 77 and 541.)

Student Notes in the margin offer just-in-time suggestions ranging from avoiding common mistakes to how to best read new notation. Conversational in tone, they give students extra explanation of the mathematics appearing on that page. (See pp. 22 and 491.)

Study Skills, ranging from time management to test preparation, appear once per section throughout the text. These suggestions for successful study habits apply to any college course and any level of student. (See pp. 181 and 224.)

Chapter Resources are additional learning materials compiled at the end of each chapter, making them easy to integrate into the course at the most appropriate time. The mathematics necessary to use the resource has been presented by the end of the section indicated with each resource.

- *Translating for Success* and *Visualizing for Success.* These are matching exercises that help students learn to translate word problems to mathematical language and to graph equations and inequalities. (See pp. 63 and 213.)
- *Collaborative Activity.* Students who work in groups generally outperform those who do not, so these optional activities direct them to explore mathematics together. Additional collaborative activities and suggestions for directing collaborative learning appear in the *Instructor's Resources Manual with Tests and Mini Lectures*. (See pp. 424 and 575.)
- *Decision Making: Connection.* Although many applications throughout the text involve decision-making situations, this feature specifically applies the math of each chapter to a context in which students may be involved in decision making. (See pp. 272 and 646.)

PRACTICE: Reinforcing Understanding

As students explore the math, they have frequent opportunities to practice, self-assess, and reinforce their understanding.

Your Turn Exercises, following every example, direct students to work a similar exercise. This provides immediate reinforcement of concepts and skills. Answers to these exercises appear at the end of each exercise set. (See pp. 75 and 393.)

New! **Check Your Understanding** offers students the chance to reflect on the concepts just discussed before beginning the exercise set. Designed to examine or extend students' understanding of one or more essential concepts of the section, this set of questions could function as an "exit ticket" after an instructional session. (See pp. 174 and 313.)

Mid-Chapter Review offers an opportunity for active review in the middle of every chapter. A brief summary of the concepts covered in the first part of the chapter is followed by two guided solutions to help students work step-by-step through solutions and a set of mixed review exercises. (See pp. 188 and 390.)

Exercise Sets

- *Vocabulary and Reading Check* exercises begin every exercise set and are designed to encourage the student to read the section. Students who can complete these exercises should be prepared to begin the remaining exercises in the exercise set. (See pp. 482 and 559.)
- *Concept Reinforcement* exercises can be true/false, matching, and/or fill-in-the-blank and appear near the beginning of many exercise sets. They are designed to build students' confidence and comprehension. Answers to all concept reinforcement exercises appear in the answer section at the back of the book. (See pp. 242 and 417.)
- *Aha!* exercises are not more difficult than neighboring exercises; in fact, they can be solved more quickly, without lengthy computation, if the student has the proper insight. They are designed to encourage students to "look before they leap." An icon indicates the first time that a new insight applies, and then it is up to the student to determine when to use that insight on subsequent exercises. (See pp. 54 and 453.)
- *Skill Review* exercises appear in every section beginning with Section 1.2. Taken together, each chapter's Skill Review exercises review all the major concepts covered in previous chapters in the text. Often these exercises focus on a single topic, such as solving equations, from multiple perspectives. (See pp. 399 and 719.)
- *Synthesis* exercises appear in each exercise set following the Skill Review exercises. Students will often need to use skills and concepts from earlier sections to solve these problems, and this will help them develop deeper insights into the current topic. The Synthesis exercises are a real strength of the text, and in the annotated instructor's edition, the authors have placed a ✓ next to selected synthesis exercises that they suggest instructors "check out" and consider assigning. These exercises may be more accessible to students than the surrounding exercises, they may extend concepts beyond the scope of the text discussion, or they may be especially beneficial in preparing students for future topics. (See pp. 244, 299, and 372–373.)
- *Writing* exercises appear just before the Skill Review exercises, and at least two more challenging exercises appear in the Synthesis exercises. Writing exercises aid student comprehension by requiring students to use critical thinking to explain concepts in one or more complete sentences. Because correct answers may vary, the only writing exercises for which answers appear at the back of the text are those in the chapter's review exercises. (See pp. 186 and 643.)
- *Quick Quizzes* with five questions appear near the end of each exercise set beginning with the second section in each chapter. Containing questions from sections already covered in the chapter, these quizzes provide a short but consistent review of the material in the chapter and help students prepare for a chapter test. (See pp. 129 and 253.)
- *Prepare to Move On* is a short set of exercises that appears at the end of every exercise set. It reviews concepts and skills previously covered in the text that will be used in the next section of the text. (See pp. 179 and 322.)

Study Summary gives students a fast and effective review of key chapter terms and concepts at the end of each chapter. Concepts are paired with worked-out examples and practice exercises for active learning and review. (See pp. 141 and 496.)

Chapter Review and Test offers a thorough chapter review, and a practice test helps to prepare students for a test covering the concepts presented in each chapter. (See pp. 349 and 649.)

Cumulative Review appears after every chapter beginning with Chapter 2 to help students retain and apply their knowledge from previous chapters. (See pp. 222 and 432.)

Acknowledgments

An outstanding team of professionals was involved in the production of this text. Judy Henn, Laurie Hurley, Helen Medley, Tamera Drozd, and Mike Penna carefully checked the book for accuracy and offered thoughtful suggestions.

Martha Morong, of Quadrata, Inc., provided editorial and production services of the highest quality, and Geri Davis, of the Davis Group, Inc., performed superb work as designer, art editor, and photo researcher. Network Graphics provided the accurate and creative illustrations and graphs.

The team at Pearson deserves special thanks. Courseware Portfolio Manager Cathy Cantin, Content Producer Ron Hampton, and Courseware Portfolio Management Assistant Alison Oehmen provided many fine suggestions, coordinated tasks and schedules, and remained involved and accessible throughout the project. Product Marketing Manager Kyle DiGiannantonio skillfully kept in touch with the needs of faculty. Director, Courseware Portfolio Management Michael Hirsch and VP, Courseware Portfolio Manager Chris Hoag deserve credit for assembling this fine team.

We thank the following professors for their thoughtful reviews and insightful comments: Shawna Haider, *Salt Lake Community College*; Ashley Nicoloff, *Glendale Community College*; and Jane Thompson, *Waubonsee Community College*

Finally, a special thank-you to all those who so generously agreed to discuss their professional use of mathematics in our chapter openers. These dedicated people all share a desire to make math more meaningful to students. We cannot imagine a finer set of role models.

M.L.B.
D.J.E.
B.L.J.

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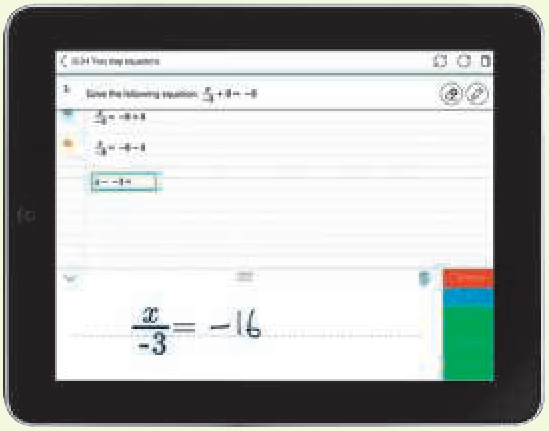

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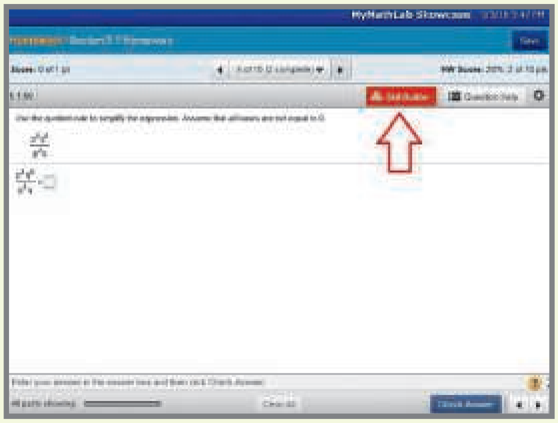

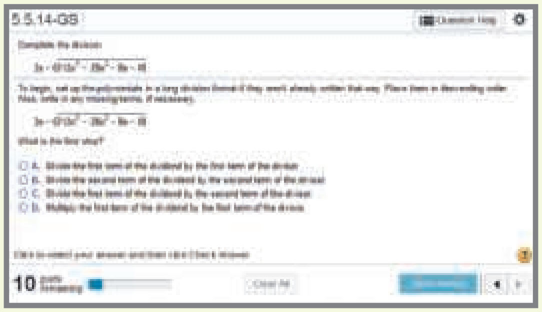

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Resources for Success

Pearson

In addition to robust course delivery, the full eText, and many assignable exercises and media assets, MyMathLab also houses the following materials to help instructors and students use this program most effectively according to his or her needs.

Student Resources

To-the-Point Objective Videos

- Concise, interactive, and objective-based videos.
- View a whole section, choose an objective, or go straight to an example.
- Interactive *Your Turn Video Check* pauses for the student to work exercises.
- Seamlessly integrated with *MyMathGuide: Notes, Practice, and Video Path.*

Chapter Test Prep Videos

- Step-by-step solutions for every problem in the Chapter Tests.
- Also available in MyMathLab

MyMathGuide: Notes, Practice, and Video Path

ISBN: 0-13-449748-1

- Guided, hands-on learning in a workbook format with space for students to show their work and record their notes and questions.
- Objective-based, correlates to the *To-the-Point Objective Videos* program.
- Highlights key concepts, skills, and definitions; offers quick reviews of key vocabulary terms with practice problems, examples with guided solutions, similar Your Turn exercises, and practice exercises with readiness checks.

Student's Solutions Manual

ISBN: 0-13-449753-8

- Contains step-by-step solutions for all odd-numbered text exercises (except the writing exercises), as well as Chapter Review, Chapter Test, and Connecting the Concepts exercises.

Instructor Resources

Annotated Instructor's Edition

ISBN: 0-13-450737-1

- Answers to all text exercises.
- Teaching tips and icons that identify writing and graphing calculator exercises.

Instructor's Solutions Manual

(download only)

ISBN: 0-13-449747-3

- Fully worked-out solutions to the odd-numbered text exercises.
- Brief solutions to the even-numbered text exercises.

Instructor's Resource Manual with Tests and Mini Lectures (download only)

ISBN: 0-13-449750-3

- Designed to help both new and adjunct faculty with course preparation and classroom management.
- Teaching tips correlated to the text by section.
- Multiple-choice and free-response chapter tests; multiple final exams.

PowerPoint® Lecture Slides

(download only)

- Editable slides present key concepts and definitions from the text.
- Also available for download through MyMathLab or via Pearsonhighered.com/IRC.

TestGen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text.

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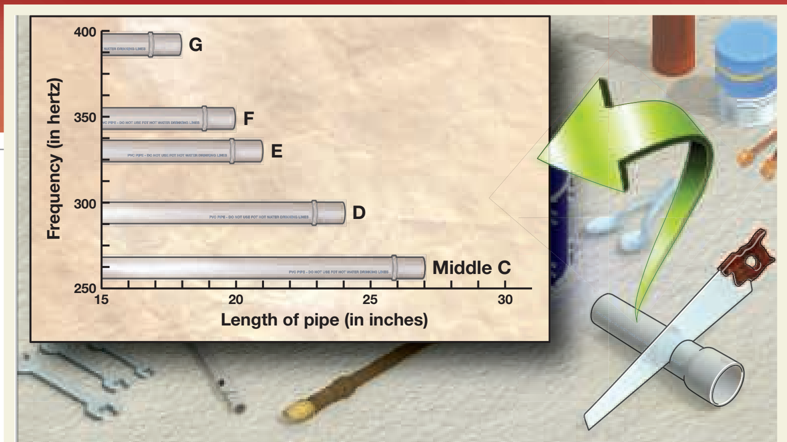
Chapter 1

Algebra and Problem Solving



Data: The Math Behind Music by NutshellEd on youtube.com, liutaiomottola.com

Make Your Own Music!

The making of music is not restricted to instruments commonly played in bands or orchestras. Saws, jugs, and pipes, among other items, have all been used to create music. In order to design an instrument, it is important that one understand the relationship between a note's pitch and the length and frequency of the wave producing the sound. The table above shows the relationship between several notes, their frequencies, and the lengths of PVC pipe that produce those sounds when struck. Instrument design and mathematics can help us understand the science of sound and the connections between music, science, and mathematics.

(See Exercise 57 in Exercise Set 1.5.)

It's true—even as a musician, I am not exempt from using math, because music is math.

Myra Flynn, a singer/songwriter from Randolph, Vermont, uses math in harmonies, time signatures, tuning systems, and all music theory. Putting an album out requires the use of even more math: calculating the number of hours worked in the studio, payments for producers and musicians, hard-copy and digital distribution regionally, and ticket and concert sales.

ALF *Active Learning Figure* Explore the math using the Active Learning Figure in MyMathLab.

SA *Student Activity* Do the Student Activity in MyMathLab to see math in action.

The principal theme of this text is problem solving in algebra. In this chapter, we begin with a review of algebraic expressions and equations. The use of algebra as part of an overall strategy for solving problems is then presented. Additional and increasing emphasis on problem solving appears throughout the book.

1.1 Some Basics of Algebra

A. Translating to Algebraic Expressions **B.** Evaluating Algebraic Expressions
C. Sets of Numbers

The primary difference between algebra and arithmetic is the use of *variables*. A letter that can be any one of various numbers is called a **variable**. If a letter always represents a particular number that never changes, it is called a **constant**. If r represents the radius of the earth, in kilometers, then r is a constant. If a represents the age of a baby chick, in minutes, then a is a variable because a changes, or *varies*, as time passes. In this text, unless stated otherwise, we assume that all letters represent variables.

An **algebraic expression** consists of variables and/or numerals, often with operation signs and grouping symbols. Some examples of algebraic expressions are:

$t + 37$; This contains the variable t, the constant 37, and the operation of addition.

$(s + t) \div 2$. This contains the variables s and t, the constant 2, grouping symbols, and the operations addition and division.

Multiplication can be written in several ways. For example, "60 times n" can be written as $60 \cdot n$, $60 \times n$, $60(n)$, $60 * n$, or simply (and usually) $60n$. Division can also be represented by a fraction bar: $\frac{9}{7}$, or $9/7$, means $9 \div 7$.

When an equals sign is placed between two expressions, an **equation** is formed. We often **solve** equations.

For example, suppose that you collect \$744 for group tickets to a concert. If you know that each ticket costs \$12, you can use an equation to determine how many tickets were purchased.

One expression for total ticket sales is 744. Another expression for total ticket sales is $12x$, where x is the number of tickets purchased. Since these are equal expressions, we can write the equation

$$12x = 744.$$

To find a **solution**, we can divide both sides of the equation by 12:

$$x = 744 \div 12 = 62.$$

Thus, 62 tickets were purchased.

Using equations to solve problems like this is a major theme of algebra.

A. Translating to Algebraic Expressions

To translate phrases to expressions, we need to know which words correspond to which operations, as shown in the following table.

Key Words

Addition	Subtraction	Multiplication	Division
add sum of plus increased by more than	subtract difference of minus decreased by less than	multiply product of times twice of	divide quotient of divided by ratio per

When the value of a number is not given, we represent that number with a variable.

Phrase	Algebraic Expression
Five *more than* some number	$n + 5$
Half *of* a number	$\frac{1}{2}t$, or $\frac{t}{2}$
Five *more than* three *times* some number	$3p + 5$
The *difference* of two numbers	$x - y$
Six *less than* the *product of* two numbers	$rs - 6$
Seventy-six percent *of* some number	$0.76z$, or $\frac{76}{100}z$

EXAMPLE 1 Translate to an algebraic expression:

Five less than forty-three percent of the quotient of two numbers.

SOLUTION We let r and s represent the two numbers.

$$(0.43)\cdot\frac{r}{s} - 5$$

Five less than forty-three percent of the quotient of two numbers

1. Translate to an algebraic expression: Half of the difference of two numbers.

YOUR TURN

Some algebraic expressions contain *exponential notation*. Many different kinds of numbers can be used as *exponents*. Here we establish the meaning of a^n when n is a counting number, 1, 2, 3, ...

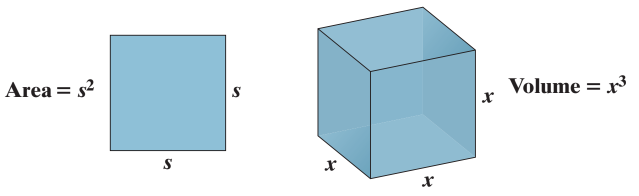
EXPONENTIAL NOTATION

The expression a^n, in which n is a counting number, means

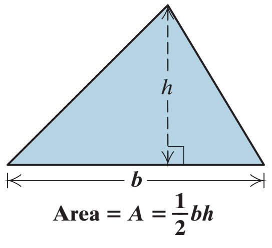
$$\underbrace{a\cdot a\cdot a\cdot\cdots\cdot a\cdot a}_{n\text{ factors}}.$$

In a^n, a is called the *base* and n is the *exponent*. When no exponent appears, the exponent is assumed to be 1. Thus, $a^1 = a$.

The expression a^n is read "a raised to the nth power" or simply "a to the nth." We read s^2 as "s-squared" and x^3 as "x-cubed." This terminology comes from the fact that the area of a square of side s is $s \cdot s = s^2$ and the volume of a cube of side x is $x \cdot x \cdot x = x^3$.



B. Evaluating Algebraic Expressions



When we replace a variable with a number, we say that we are **substituting** for the variable. The calculation that follows the substitution is called **evaluating the expression**.

Geometric formulas are often evaluated. In the following example, we use the formula for the area of a triangle with a base of length b and a height of length h.

EXAMPLE 2 The base of a triangular sail is 3.1 m and the height is 4 m. Find the area of the sail.

SOLUTION We substitute 3.1 for b and 4 for h and multiply to evaluate the expression:

$$\begin{aligned} \tfrac{1}{2} \cdot b \cdot h &= \tfrac{1}{2} \cdot 3.1 \cdot 4 \\ &= 6.2 \text{ square meters (sq m or m}^2\text{).} \end{aligned}$$

2. The base of a triangle is 5 ft and the height is 3 ft. Find the area of the triangle.

YOUR TURN

Exponential notation tells us that 5^2 means $5 \cdot 5$, or 25, but what does $1 + 2 \cdot 5^2$ mean? If we add 1 and 2 and multiply by 25, we get 75. If we multiply 2 times 5^2 and add 1, we get 51. A third possibility is to square $2 \cdot 5$ to get 100 and then add 1. The following convention indicates that only the second of these approaches is correct: We square 5, then multiply, and then add.

Student Notes

Step (3) states that when division precedes multiplication, the division is performed first. Thus, $20 \div 5 \cdot 2$ represents $4 \cdot 2$, or 8. Similarly, $9 - 3 + 1$ represents $6 + 1$, or 7.

RULES FOR ORDER OF OPERATIONS

1. Simplify within any grouping symbols such as ( ), [ ], { }, working in the innermost symbols first.
2. Simplify all exponential expressions.
3. Perform all multiplication and division, working from left to right.
4. Perform all addition and subtraction, working from left to right.

EXAMPLE 3 Evaluate $5 + 2(a - 1)^2$ for $a = 4$.

SOLUTION

$$\begin{aligned} 5 + 2(a-1)^2 &= 5 + 2(4-1)^2 && \text{Substituting} \\ &= 5 + 2(3)^2 && \text{Working within parentheses first} \\ &= 5 + 2(9) && \text{Simplifying } 3^2 \\ &= 5 + 18 && \text{Multiplying} \\ &= 23 && \text{Adding} \end{aligned}$$

YOUR TURN

3. Evaluate $2(x + 1)^2 - 10$ for $x = 5$.

Step (3) in the rules for order of operations tells us to divide before we multiply when division appears first, reading left to right. This means that an expression like $6 \div 2x$ means $(6 \div 2)x$.

CAUTION!
$6 \div 2x = (6 \div 2)x$,

$6 \div (2x) = \dfrac{6}{2x}$,

$6 \div 2x$ does not mean $6 \div (2x)$.

EXAMPLE 4 Evaluate $9 - x^3 + 6 \div 2y^2$ for $x = 2$ and $y = 5$.

SOLUTION

$$\begin{aligned} 9 - x^3 + 6 \div 2y^2 &= 9 - 2^3 + 6 \div 2(5)^2 && \text{Substituting} \\ &= 9 - 8 + 6 \div 2 \cdot 25 && \text{Simplifying } 2^3 \text{ and } 5^2 \\ &= 9 - 8 + 3 \cdot 25 && \text{Dividing} \\ &= 9 - 8 + 75 && \text{Multiplying} \\ &= 1 + 75 && \text{Subtracting} \\ &= 76 && \text{Adding} \end{aligned}$$

YOUR TURN

4. Evaluate $8a^2 \div 5b - 4 + a$ for $a = 5$ and $b = 2$.

C. Sets of Numbers

When evaluating algebraic expressions, and in problem solving in general, we often must examine the *type* of numbers used. For example, if a formula is used to determine an optimal class size, fractions must be rounded up or down, since it is impossible to have a fraction part of a student. Three frequently used sets of numbers are listed below.

NATURAL NUMBERS, WHOLE NUMBERS, AND INTEGERS

Natural Numbers (or Counting Numbers)

Those numbers used for counting: $\{1, 2, 3, \ldots\}$

Whole Numbers

The set of natural numbers with 0 included: $\{0, 1, 2, 3, \ldots\}$

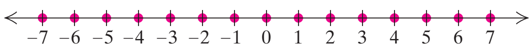
Integers

The set of all whole numbers and their opposites:

$$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

The dots are called ellipses and indicate that the pattern continues without end.

Integers correspond to the points on the number line as follows:



The set containing the numbers -2, 1, and 3 can be written $\{-2, 1, 3\}$. This set is written using **roster notation**, in which all members of a set are listed. Roster notation was used for the three sets listed above. A second type of set notation,

Check Your UNDERSTANDING

Choose from the following expressions an appropriate algebraic translation of each phrase.

a) $0.06x + 1$
b) $x + y - 6$
c) $3(x + y)$
d) $2(x - y)$
e) $\frac{1}{3}x$
f) $\dfrac{x}{y} - 3$

1. One-third of a number
2. Six less than the sum of two numbers
3. Twice the difference of two numbers
4. One more than six percent of a number
5. Three less than the quotient of two numbers
6. The product of three and the sum of two numbers

set-builder notation, specifies conditions under which a number is in the set. The following example of set-builder notation is read as shown:

$$\underbrace{\{x}_{\text{"The set of all } x} \quad \underbrace{|}_{\text{such that}} \quad \underbrace{x \text{ is a number between 1 and 5}\}}_{x \text{ is a number between 1 and 5"}}$$

Set-builder notation is generally used when it is difficult to list a set using roster notation.

EXAMPLE 5 Using both roster notation and set-builder notation, represent the set consisting of the first 15 even natural numbers.

SOLUTION

Using roster notation: $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30\}$

Using set-builder notation: $\{n \mid n \text{ is an even number between 1 and } 31\}$

Note that other descriptions of the set are possible. For example, $\{2x \mid x$ is an integer *and* $1 \leq x \leq 15\}$ is a common way of writing this set.

5. Using both roster notation and set-builder notation, represent the set of all multiples of 5 between 1 and 21.

YOUR TURN

The symbol $\in$ is used to indicate that an **element** or a **member** belongs to a set. Thus if $A = \{2, 4, 6, 8\}$, we can write $4 \in A$ to indicate that 4 *is an element of* A. We can also write $5 \notin A$ to indicate that 5 *is not an element of* A.

EXAMPLE 6 Classify the statement $8 \in \{x \mid x \text{ is an integer}\}$ as either true or false.

SOLUTION Since 8 *is* an integer, the statement is true. In other words, since 8 is an integer, it belongs to the set of all integers.

6. Classify the statement $\frac{1}{2} \in \{x \mid x \text{ is a whole number}\}$ as either true or false.

YOUR TURN

Using set-builder notation, we can describe the set of all *rational numbers*.

RATIONAL NUMBERS

Numbers that can be expressed as an integer divided by a nonzero integer are called *rational numbers*:

$$\left\{\frac{p}{q} \,\middle|\, p \text{ is an integer, } q \text{ is an integer, and } q \neq 0\right\}.$$

Rational numbers can be written using fraction notation or decimal notation. *Fraction notation* uses symbolism like the following:

$$\frac{5}{8}, \quad \frac{12}{-7}, \quad \frac{-17}{15}, \quad -\frac{9}{7}, \quad \frac{39}{1}, \quad \frac{0}{6}.$$

In *decimal notation*, rational numbers either *terminate* (end) or *repeat* a block of digits.

For example, decimal notation for $\frac{5}{8}$ terminates, since $\frac{5}{8}$ means $5 \div 8$, and long division shows that $\frac{5}{8} = 0.625$, a decimal that ends, or terminates.

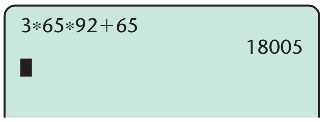
On the other hand, decimal notation for $\frac{6}{11}$ repeats, since $6 \div 11 = 0.5454\ldots$, a repeating decimal. Repeating decimal notation can be abbreviated by writing a bar over the repeating part—in this case, $0.\overline{54}$.

Technology Connection

Technology Connections are activities that make use of features that are common to most graphing calculators. Students may consult a user's manual for exact keystrokes. Most graphing calculators share the following characteristics.

Screen. The large screen can show graphs and tables as well as the expressions entered. Computations are performed in the **home screen**. On many calculators, the home screen is accessed by pressing 2ND QUIT. The **cursor** shows location on the screen, and the **contrast** (set by 2ND ⌃ or 2ND ⌄) determines how dark the characters appear.

Keypad. To access options written above the keys, we press 2ND or ALPHA and then the key. Expressions are generally entered as they would appear in print. For example, to evaluate $3xy + x$ for $x = 65$ and $y = 92$, we press 3 × 65 × 92 + 65 and then ENTER. The value of the expression, 18005, will appear at the right of the screen.



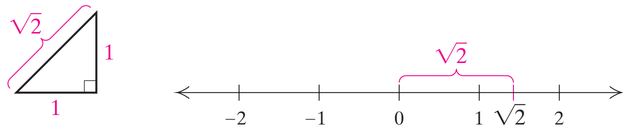
Evaluate each of the following.

1. $27a - 18b$, for $a = 136$ and $b = 13$
2. $19xy - 9x + 13y$, for $x = 87$ and $y = 29$

VIDEO

Many numbers, like π, $\sqrt{2}$, and $-\sqrt{15}$, are not rational numbers. For example, $\sqrt{2}$ is the number for which $\sqrt{2} \cdot \sqrt{2} = 2$. A calculator's representation of $\sqrt{2}$ as 1.414213562 is an approximation since $(1.414213562)^2$ is not exactly 2.

To see that $\sqrt{2}$ is a "real" point on the number line, we can show that when a right triangle has two legs of length 1, the remaining side has length $\sqrt{2}$. Thus we can "measure" $\sqrt{2}$ units and locate $\sqrt{2}$ on the number line.



Numbers like π, $\sqrt{2}$, and $-\sqrt{15}$ are said to be **irrational**. Decimal notation for irrational numbers neither terminates nor repeats.

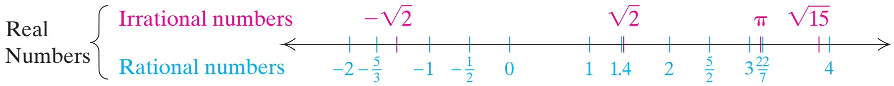
The set of all rational numbers, combined with the set of all irrational numbers, gives us the set of all **real numbers**.

REAL NUMBERS

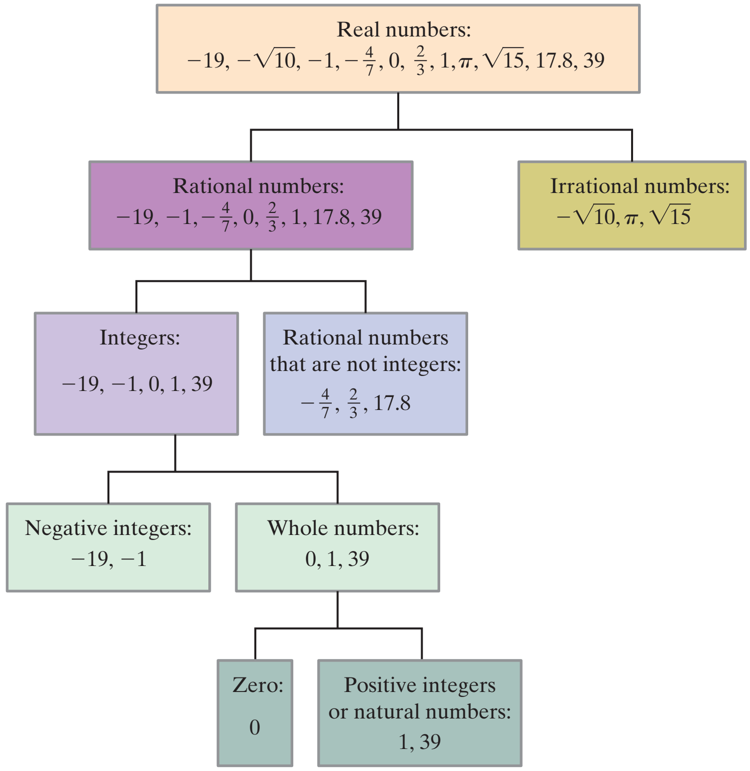
Numbers that are either rational or irrational are called *real numbers*:

$$\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$$

Every point on the number line represents some real number, and every real number is represented by some point on the number line.



The following figure shows the relationships among various kinds of numbers, along with examples of how real numbers can be sorted.



EXAMPLE 7 Which numbers in the following list are **(a)** whole numbers? **(b)** integers? **(c)** rational numbers? **(d)** irrational numbers? **(e)** real numbers?

$$-29, \quad -\tfrac{7}{4}, \quad 0, \quad 2, \quad 3.9, \quad \sqrt{42}, \quad 78$$

SOLUTION

a) 0, 2, and 78 are whole numbers.

b) -29, 0, 2, and 78 are integers.

c) -29, $-\frac{7}{4}$, 0, 2, 3.9, and 78 are rational numbers.

d) $\sqrt{42}$ is an irrational number.

e) -29, $-\frac{7}{4}$, 0, 2, 3.9, $\sqrt{42}$, and 78 are all real numbers.

YOUR TURN

7. Which numbers in the following list are integers?

$$-245, \quad 0, \quad 15, \quad \sqrt{11}, \quad \tfrac{2}{3}$$

When every member of one set is a member of a second set, the first set is a **subset** of the second set. Thus if $A = \{2, 4, 6\}$ and $B = \{1, 2, 4, 5, 6\}$, we write $A \subseteq B$ to indicate that A *is a subset of* B. Similarly, if $\mathbb{N}$ represents the set of all natural numbers and $\mathbb{Z}$ is the set of all integers, we can write $\mathbb{N} \subseteq \mathbb{Z}$. Additional statements can be made using other sets in the diagram above.

Study Skills

Get the Facts

Throughout this textbook, you will find a feature called *Study Skills*. These tips are intended to help improve your math study skills. On the first day of class, we recommend that you collect the course information shown here.

Instructor:
Name ______________________
Office hours and location ______________________
Phone number ______________________
E-mail address ______________________

Classmates:
1. Name ______________________
Phone number ______________________
E-mail address ______________________
2. Name ______________________
Phone number ______________________
E-mail address ______________________

Math lab on campus:
Location ______________________
Hours ______________________
Phone number ______________________
E-mail address ______________________

Tutoring:
Campus location ______________________
Hours ______________________
E-mail address ______________________

Important supplements:
(See the preface for a complete list of available supplements.)
Supplements recommended by the instructor.
____
____

1.1 EXERCISE SET

FOR EXTRA HELP MyMathLab®

Vocabulary and Reading Check

Choose from the following list the word or words that best complete each statement.

base	exponent	terminating
constant	irrational	value
division	rational	variable
evaluating	repeating	

1. A letter that can be any one of a set of numbers is called a(n) ______________.

2. A letter representing a specific number that never changes is called a(n) ______________.

3. When $x = 10$, the ______________ of the expression $4x$ is 40.

4. In a^b, the letter a is called the ______________ and the letter b is called the ______________.

5. When all variables in a variable expression are replaced by numbers and a result is calculated, we say that we are ______________ the expression.

6. To calculate $4 + 12 \div 3 \cdot 2$, the first operation that we perform is ______________.

7. A number that can be written in the form a/b, where a and b are integers (with $b \neq 0$), is said to be a(n) ______________ number.

8. A real number that cannot be written as a quotient of two integers is an example of a(n) ______________ number.

9. Division can be used to show that $\frac{7}{40}$ can be written as a(n) ______________ decimal.

10. Division can be used to show that $\frac{13}{7}$ can be written as a(n) ______________ decimal.

A. Translating to Algebraic Expressions

Use mathematical symbols to translate each phrase.

11. Five less than some number
12. Ten more than some number
13. Twice a number
14. Eight times a number
15. Twenty-nine percent of some number
16. Thirteen percent of some number
17. Six less than half of a number
18. Three more than twice a number
19. Seven more than ten percent of some number
20. Four less than six percent of some number
21. One less than the product of two numbers
22. One more than the difference of two numbers
23. Ninety miles per every four gallons of gas
24. One hundred words per every sixty seconds

B. Evaluating Algebraic Expressions

In Exercises 25–28, find the area of a square flower garden with the given length of a side. Use $A = s^2$.

25. Side = 6 ft
26. Side = 12 ft
27. Side = 0.5 m
28. Side = 2.5 m

In Exercises 29–32, find the area of a triangular fireplace with the given base and height. Use $A = \frac{1}{2}bh$.

29. Base = 5 ft, height = 7 ft
30. Base = 2.9 m, height = 2.1 m
31. Base = 7 ft, height = 3.2 ft
32. Base = 3.6 ft, height = 4 ft

To the student and the instructor: *Throughout this text, selected exercises are marked with the icon* **Aha!**. *Students who pause to inspect an Aha! exercise should find the answer more readily than those who proceed mechanically. This may involve looking at an earlier exercise or example, or performing calculations in a more efficient manner. Some Aha! exercises are left unmarked to encourage students to always pause before working a problem.*

Evaluate each expression using the values provided.

33. $3(x - 7) + 2$, for $x = 10$
34. $5 + (2x - 3)$, for $x = 8$
35. $12 + 3(n + 2)^2$, for $n = 1$
36. $(n - 10)^2 - 8$, for $n = 15$
37. $4x + y$, for $x = 2$ and $y = 3$
38. $8a - b$, for $a = 5$ and $b = 7$
39. $20 + r^2 - s$, for $r = 5$ and $s = 10$
40. $m^3 + 7 - n$, for $m = 2$ and $n = 8$
41. $2c \div 3b$, for $b = 2$ and $c = 6$
42. $3z \div 2y$, for $y = 1$ and $z = 6$
43. **Aha!** $3n^2p - 3pn^2$, for $n = 5$ and $p = 9$
44. $2a^3b - 2b^2$, for $a = 3$ and $b = 7$
45. $5x \div (2 + x - y)$, for $x = 6$ and $y = 2$
46. $3(m + 2n) \div m$, for $m = 7$ and $n = 0$
47. $[10 - (a - b)]^2$, for $a = 7$ and $b = 2$
48. $[17 - (x + y)]^2$, for $x = 4$ and $y = 1$