# Intermediate ALGEBRAA TENTH EDITION

# Bittinger | Ellenbogen | Johnson



# **Resources for Success**

## MyMathLab<sup>®</sup> Online Course

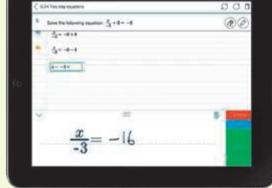
The course for Intermediate Algebra: Concepts and Applications, 10th Edition, includes all of MyMathLab's robust features and functionality, plus these additional highlights.

### New! Workspace

Workspace Assignments allow students to work through an exercise step by step, showing their mathematical reasoning. Students receive immediate feedback after they complete each step, and helpful hints and videos are available for guidance, as needed. When students access Workspace using a mobile device, handwriting-recognition software allows them to write out answers naturally using their fingertip or a stylus.



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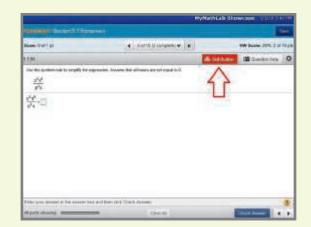


### New! Learning Catalytics

Learning Catalytics uses students' mobile devices for an engagement, assessment, and classroom intelligence system that gives instructors real-time feedback on student learning.

### New! Skill Builder Adaptive Practice

When a student struggles with assigned homework, Skill Builder exercises offer just-in-time additional adaptive practice. The adaptive engine tracks student performance and delivers questions to each individual that adapt to his or her level of understanding. When the system has determined that the student has a high probability of successfully completing the assigned exercise, it suggests that the student return to the assignment. When Skill Builder is enabled for an assignment, students can choose to do the extra practice without being prompted. This new feature allows instructors to assign fewer questions for homework so that students can complete as many or as few questions as needed.



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### **Interactive Exercises**

MyMathLab's hallmark interactive exercises help build problem-solving skills and foster conceptual understanding. For this seventh edition, Guided Solutions exercises were added to Mid-Chapter Reviews to reinforce the step-by-step problemsolving process, while the new Drag & Drop functionality was applied to matching exercises throughout the course to better assess a student's understanding of the concepts.

### www.mymathlab.com

# **Intermediate Algebra**

## **Concepts and Applications**

## **Tenth Edition**

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### A KEY TO THE ICONS IN THE EXERCISE SETS

4	Concept reinforcement exercises, indi- cated by blue exercise numbers, provide basic practice with the new concepts and vocabulary.
Aha!	Exercises labeled Ana! indicate the first time that a new insight can greatly simplify a problem and help students be alert to using that insight on following exercises. They are not more difficult.
	Calculator exercises are designed to be worked using either a scientific calculator or a graphing calculator.
$\sim$	Graphing calculator exercises are designed to be worked using a graphing calculator and often provide practice for concepts discussed in the Technology Connections.
Ċ	Writing exercises are designed to be answered using one or more complete sentences.
1	A check mark in the annotated instructor's edition indicates Synthesis exercises that the authors consider particularly beneficial for students.
4	The research icon indicates an exercise in which students are asked to use research skills to extend or to explore further applications from the text.

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# Preface

Welcome to the tenth edition of Intermediate Algebra: Concepts and Applications, one of three programs in an algebra series that also includes Elementary and Intermediate Algebra: Concepts and Applications, Seventh Edition, and Elementary Algebra: Concepts and Applications, Tenth Edition. As always, our goal is to present the content of the course clearly yet with enough depth to allow success in future courses. You will recognize many proven features, applications, and explanations; you will also find new material developed as a result of our experience in the classroom as well as from insights from faculty and students.

### **Understanding and Applying Concepts**

Our goal is to help today's students learn and retain mathematical concepts. To achieve this, we feel that we must prepare students in developmental mathematics for the transition from "skills-oriented" elementary algebra courses to more "concept-oriented" college-level mathematics courses. This requires the development of critical thinking skills: to reason mathematically, to communicate mathematically, and to identify and solve mathematical problems.

Following are aspects of our approach that we use to help meet the challenges we all face when teaching developmental mathematics.

**Problem** We use problem solving and applications to motivate the students wherever possible, and we **Solving** include real-life applications and problem-solving techniques throughout the text. Problem solving encourages students to think about how mathematics can be used, and it helps to prepare them for more advanced material in future courses.

In Chapter 1, we introduce our five-step process for solving problems: (1) Familiarize, (2) Translate, (3) Carry out, (4) Check, and (5) State the answer. Repeated use of this problemsolving strategy throughout the text provides students with a starting point for any type of problem they encounter, and frees them to focus on the unique aspects of the particular problem. We often use estimation and carefully checked guesses to help with the Familiarize and Check steps (see pp. 169 and 394).

Applications Interesting, contemporary applications of mathematics, many of which make use of real data, help motivate students and instructors. In this new edition, we have updated real-world data examples and exercises to include subjects such as website design (p. 123), college readiness (p. 195), and bald eagles (p. 636). For a complete list of applications and the page numbers on which they can be found, please refer to the Index of Applications at the back of the book.

**Conceptual** Growth in mathematical ability includes not only mastering skills and procedures but also deepening Understanding understanding of mathematical concepts. We are careful to explain the reasoning and the principles behind procedures and to use accurate mathematical terminology in our discussion. In addition, we provide a variety of opportunities for students to develop their understanding of mathematical concepts, including making connections between concepts, learning through active exploration, applying and extending concepts, using new vocabulary, communicating comprehension through writing, and employing research skills to extend their examination of a topic.

### **Guided Learning Path**

To enhance the learning process and improve learner outcomes, our program provides a broad range of support for students and instructors. Each person can personalize his or her learning or teaching experience by accessing help when he or she needs it.

### **PREPARE:** Studying the Concepts

Students can learn about each math concept by reading the textbook or etext, watching the To-the-Point Objective videos, participating in class, working in the *MyMathGuide* workbook— or using whatever combination of these course resources works best for him or her.

Enhanced!

**Text** The exposition, examples, and exercises have been carefully reviewed and, as appropriate, revised or replaced. New features (see below) include more systematic review and preparation for practice, as well as stronger focus on the real-world applications for the math.



**MyMathLab** has been greatly expanded for this course, including adding more ways for students to personalize their learning path so they can effectively study, master, and retain the math. (See pp. xiv–xv for more details.)

**To-the-Point Objective Videos** is a comprehensive program of objective-based, interactive videos that can be used hand-in-hand with the *MyMathGuide* workbook. Video support for Interactive Your Turn exercises in the videos prompts students to solve problems and receive instant feedback.

*MyMathGuide: Notes, Practice, and Video Path* is an objective-based workbook (available in print and in MyMathLab) for guided, hands-on learning. It offers vocabulary, skill, and concept review; and problem-solving practice with space for students to fill in the answers and stepped-out solutions to problems, show their work, and write notes. Students can use *MyMathGuide*—while watching the videos, listening to the instructor's lecture, or reading the textbook or etext—to reinforce and self-assess their learning.

### PARTICIPATE: Making Connections through Active Exploration

Knowing that developing a solid grasp of the big picture is a key to student success, we offer many opportunities for active learning to help students practice, review, and confirm their understanding of key concepts and skills.

New! Chapter Opener Applications with Infographics use current data and applications to present the math in context. Each application is related to exercises in the text to help students model, visualize, learn, *and* retain the math. We also added many new spotlights on real people sharing how they use math in their careers.

**Algebraic–Graphical Connections,** which appear occasionally throughout the text, draw explicit connections between the algebra and the corresponding graphical visualizations. (See pp. 154 and 504.)

**Exploring the Concept,** appearing once in nearly every chapter, encourages students to think about or visualize a key mathematical concept. (See pp. 171 and 480.) These activities lead into the **Active Learning Figure** interactive animations available in MyMathLab. Students can manipulate Active Learning Figures through guided and open-ended exploration to further solidify their understanding of these concepts.

**Connecting the Concepts** summarizes concepts from several sections or chapters and illustrates connections between them. Appearing at least once in every chapter, this feature includes a set of mixed exercises to help students make these connections. (See pp. 261 and 339.)

**Technology Connection** is an optional feature in each chapter that helps students use a graphing calculator or a graphing calculator app to visualize concepts. Exercises are included with many of these features, and additional exercises in many exercise sets are marked with a graphing calculator icon to indicate more practice with this optional use of technology. (See pp. 77 and 541.)

**Student Notes** in the margin offer just-in-time suggestions ranging from avoiding common mistakes to how to best read new notation. Conversational in tone, they give students extra explanation of the mathematics appearing on that page. (See pp. 22 and 491.)

**Study Skills,** ranging from time management to test preparation, appear once per section throughout the text. These suggestions for successful study habits apply to any college course and any level of student. (See pp. 181 and 224.)

**Chapter Resources** are additional learning materials compiled at the end of each chapter, making them easy to integrate into the course at the most appropriate time. The mathematics necessary to use the resource has been presented by the end of the section indicated with each resource.

- *Translating for Success* and *Visualizing for Success*. These are matching exercises that help students learn to translate word problems to mathematical language and to graph equations and inequalities. (See pp. 63 and 213.)
- *Collaborative Activity.* Students who work in groups generally outperform those who do not, so these optional activities direct them to explore mathematics together. Additional collaborative activities and suggestions for directing collaborative learning appear in the *Instructor's Resources Manual with Tests and Mini Lectures.* (See pp. 424 and 575.)
- *Decision Making: Connection.* Although many applications throughout the text involve decision-making situations, this feature specifically applies the math of each chapter to a context in which students may be involved in decision making. (See pp. 272 and 646.)

### PRACTICE: Reinforcing Understanding

As students explore the math, they have frequent opportunities to practice, self-assess, and reinforce their understanding.

**Your Turn Exercises,** following every example, direct students to work a similar exercise. This provides immediate reinforcement of concepts and skills. Answers to these exercises appear at the end of each exercise set. (See pp. 75 and 393.)

New! Check Your Understanding offers students the chance to reflect on the concepts just discussed before beginning the exercise set. Designed to examine or extend students' understanding of one or more essential concepts of the section, this set of questions could function as an "exit ticket" after an instructional session. (See pp. 174 and 313.)

**Mid-Chapter Review** offers an opportunity for active review in the middle of every chapter. A brief summary of the concepts covered in the first part of the chapter is followed by two guided solutions to help students work step-by-step through solutions and a set of mixed review exercises. (See pp. 188 and 390.)

### **Exercise Sets**

- *Vocabulary and Reading Check* exercises begin every exercise set and are designed to encourage the student to read the section. Students who can complete these exercises should be prepared to begin the remaining exercises in the exercise set. (See pp. 482 and 559.)
- *Concept Reinforcement* exercises can be true/false, matching, and/or fill-in-the-blank and appear near the beginning of many exercise sets. They are designed to build students' confidence and comprehension. Answers to all concept reinforcement exercises appear in the answer section at the back of the book. (See pp. 242 and 417)
- *Aha!* exercises are not more difficult than neighboring exercises; in fact, they can be solved more quickly, without lengthy computation, if the student has the proper insight. They are designed to encourage students to "look before they leap." An icon indicates the first time that a new insight applies, and then it is up to the student to determine when to use that insight on subsequent exercises. (See pp. 54 and 453.)
- *Skill Review* exercises appear in every section beginning with Section 1.2. Taken together, each chapter's Skill Review exercises review all the major concepts covered in previous chapters in the text. Often these exercises focus on a single topic, such as solving equations, from multiple perspectives. (See pp. 399 and 719.)
- Synthesis exercises appear in each exercise set following the Skill Review exercises. Students will often need to use skills and concepts from earlier sections to solve these problems, and this will help them develop deeper insights into the current topic. The Synthesis exercises are a real strength of the text, and in the annotated instructor's edition, the authors have placed a next to selected synthesis exercises that they suggest instructors "check out" and consider assigning. These exercises may be more accessible to students than the surrounding exercises, they may extend concepts beyond the scope of the text discussion, or they may be especially beneficial in preparing students for future topics. (See pp. 244, 299, and 372–373.)
- *Writing* exercises appear just before the Skill Review exercises, and at least two more challenging exercises appear in the Synthesis exercises. Writing exercises aid student comprehension by requiring students to use critical thinking to explain concepts in one or more complete sentences. Because correct answers may vary, the only writing exercises for which answers appear at the back of the text are those in the chapter's review exercises. (See pp. 186 and 643.)
- *Quick Quizzes* with five questions appear near the end of each exercise set beginning with the second section in each chapter. Containing questions from sections already covered in the chapter, these quizzes provide a short but consistent review of the material in the chapter and help students prepare for a chapter test. (See pp. 129 and 253.)
- *Prepare to Move On* is a short set of exercises that appears at the end of every exercise set. It reviews concepts and skills previously covered in the text that will be used in the next section of the text. (See pp. 179 and 322.)

**Study Summary** gives students a fast and effective review of key chapter terms and concepts at the end of each chapter. Concepts are paired with worked-out examples and practice exercises for active learning and review. (See pp. 141 and 496.)

**Chapter Review and Test** offers a thorough chapter review, and a practice test helps to prepare students for a test covering the concepts presented in each chapter. (See pp. 349 and 649.)

**Cumulative Review** appears after every chapter beginning with Chapter 2 to help students retain and apply their knowledge from previous chapters. (See pp. 222 and 432.)

### Acknowledgments

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Finally, a special thank-you to all those who so generously agreed to discuss their professional use of mathematics in our chapter openers. These dedicated people all share a desire to make math more meaningful to students. We cannot imagine a finer set of role models.

> M.L.B. D.J.E. B.L.J.

# **Resources for Success**

# Pearson

## MyMathLab® Online Course

The course for *Intermediate Algebra: Concepts and Applications,* 10th Edition, includes all of MyMathLab's robust features and functionality, plus these additional highlights.

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### New! Learning Catalytics

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### **Interactive Exercises**

MyMathLab's hallmark interactive exercises help build problem-solving skills and foster conceptual understanding. For this seventh edition, Guided Solutions exercises were added to Mid-Chapter Reviews to reinforce the step-by-step problemsolving process, while the *new* Drag & Drop functionality was applied to matching exercises throughout the course to better assess a student's understanding of the concepts.

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- View a whole section, choose an objective, or go straight to an example.
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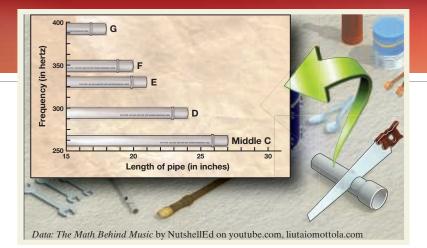
- Editable slides present key concepts and definitions from the text.
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Chapter

# Algebra and Problem Solving



he making of music is not restricted to instruments commonly played in bands or orchestras. Saws, jugs, and pipes, among other items, have all been used to create music. In order to design an instrument, it is important that one understand the relationship between a note's pitch and the length and frequency of the wave producing the sound. The table above shows the relationship between several notes, their frequencies, and the lengths of PVC pipe that produce those sounds when struck. Instrument design and mathematics can help us understand the science of sound and the connections between music, science, and mathematics.

(See Exercise 57 in Exercise Set 1.5.)

## It's true—even as a musician, I am not exempt from using math, because music is math.

Myra Flynn, a singer/songwriter from Randolph, Vermont, uses math in harmonies, time signatures, tuning systems, and all music theory. Putting an album out requires the use of even more math: calculating the number of hours worked in the studio, payments for producers and musicians, hard-copy and digital distribution regionally, and ticket and concert sales.

## Make Your Own Music!

- **1.1** Some Basics of Algebra
- 1.2 Operations and Properties of Real Numbers
- **1.3** Solving Equations

### **MID-CHAPTER REVIEW**

- 1.4 Introduction to Problem Solving
- **1.5** Formulas, Models, and Geometry

### **CONNECTING THE CONCEPTS**

- **1.6** Properties of Exponents
- 1.7 Scientific Notation

### **CHAPTER RESOURCES**

Translating for Success Collaborative Activity Decision Making: Connection

### STUDY SUMMARY

### REVIEW EXERCISES CHAPTER TEST





The principal theme of this text is problem solving in algebra. In this chapter, we begin with a review of algebraic expressions and equations. The use of algebra as part of an overall strategy for solving problems is then presented. Additional and increasing emphasis on problem solving appears throughout the book.

## 1.1

## Some Basics of Algebra

A. Translating to Algebraic Expressions
 B. Evaluating Algebraic Expressions
 C. Sets of Numbers

The primary difference between algebra and arithmetic is the use of *variables*. A letter that can be any one of various numbers is called a **variable**. If a letter always represents a particular number that never changes, it is called a **constant**. If r represents the radius of the earth, in kilometers, then r is a constant. If a represents the age of a baby chick, in minutes, then a is a variable because a changes, or *varies*, as time passes. In this text, unless stated otherwise, we assume that all letters represent variables.

An **algebraic expression** consists of variables and/or numerals, often with operation signs and grouping symbols. Some examples of algebraic expressions are:

t + 37;	This contains the variable <i>t</i> , the constant 37, and the operation of addition.
$(s+t) \div 2.$	This contains the variables <i>s</i> and <i>t</i> , the constant 2, grouping symbols, and the operations addition and division.

Multiplication can be written in several ways. For example, "60 times *n*" can be written as  $60 \cdot n$ ,  $60 \times n$ , 60(n), 60 \* n, or simply (and usually) 60n. Division can also be represented by a fraction bar:  $\frac{9}{7}$ , or 9/7, means  $9 \div 7$ .

When an equals sign is placed between two expressions, an **equation** is formed. We often **solve** equations.

For example, suppose that you collect \$744 for group tickets to a concert. If you know that each ticket costs \$12, you can use an equation to determine how many tickets were purchased.

One expression for total ticket sales is 744. Another expression for total ticket sales is 12x, where x is the number of tickets purchased. Since these are equal expressions, we can write the equation

12x = 744.

To find a solution, we can divide both sides of the equation by 12:

 $x = 744 \div 12 = 62.$ 

Thus, 62 tickets were purchased.

Using equations to solve problems like this is a major theme of algebra.

### A. Translating to Algebraic Expressions

To translate phrases to expressions, we need to know which words correspond to which operations, as shown in the following table.

### **Key Words**

Addition	Subtraction	Multiplication	Division
add	subtract	multiply	divide
sum of	difference of	product of	quotient of
plus	minus	times	divided by
increased by	decreased by	twice	ratio
more than	less than	of	per

When the value of a number is not given, we represent that number with a variable.

Phrase	Algebraic Expression
Five more than some number	n + 5
Half <i>of</i> a number	$\frac{1}{2}t$ , or $\frac{t}{2}$
Five <i>more than</i> three <i>times</i> some number The <i>difference</i> of two numbers Six <i>less than</i> the <i>product of</i> two numbers Seventy-six percent <i>of</i> some number	3p + 5 x - y rs - 6 $0.76z$ , or $\frac{76}{100}z$

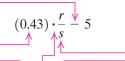
**EXAMPLE 1** Translate to an algebraic expression:

Five less than forty-three percent of the quotient of two numbers.

**SOLUTION** We let *r* and *s* represent the two numbers.

**1.** Translate to an algebraic expression: Half of the difference of two numbers.





Five less than forty-three percent of the quotient of two numbers YOUR TURN

Some algebraic expressions contain exponential notation. Many different kinds of numbers can be used as *exponents*. Here we establish the meaning of  $a^n$ when n is a counting number,  $1, 2, 3, \ldots$ .

### **EXPONENTIAL NOTATION**

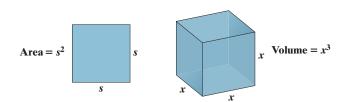
The expression  $a^n$ , in which *n* is a counting number, means

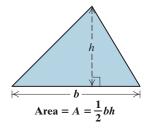
$$a \cdot a \cdot a \cdot \cdots \cdot a \cdot a$$

*n* factors

In  $a^n$ , a is called the *base* and n is the *exponent*. When no exponent appears, the exponent is assumed to be 1. Thus,  $a^1 = a$ .

The expression  $a^n$  is read "a raised to the *n*th power" or simply "a to the *n*th." We read  $s^2$  as "s-squared" and  $x^3$  as "x-cubed." This terminology comes from the fact that the area of a square of side s is  $s \cdot s = s^2$  and the volume of a cube of side x is  $x \cdot x \cdot x = x^3$ .





### **B. Evaluating Algebraic Expressions**

When we replace a variable with a number, we say that we are substituting for the variable. The calculation that follows the substitution is called evaluating the expression.

Geometric formulas are often evaluated. In the following example, we use the formula for the area of a triangle with a base of length b and a height of length h.

**EXAMPLE 2** The base of a triangular sail is 3.1 m and the height is 4 m. Find the area of the sail.



**SOLUTION** We substitute 3.1 for b and 4 for h and multiply to evaluate the expression:

 $\frac{1}{2} \cdot b \cdot h = \frac{1}{2} \cdot 3.1 \cdot 4$ 

= 6.2 square meters (sq m or m<sup>2</sup>).

### YOUR TURN

Exponential notation tells us that  $5^2$  means  $5 \cdot 5$ , or 25, but what does  $1 + 2 \cdot 5^2$  mean? If we add 1 and 2 and multiply by 25, we get 75. If we multiply 2 times  $5^2$  and add 1, we get 51. A third possibility is to square  $2 \cdot 5$  to get 100 and then add 1. The following convention indicates that only the second of these approaches is correct: We square 5, then multiply, and then add.

### **RULES FOR ORDER OF OPERATIONS**

- **1.** Simplify within any grouping symbols such as ( ), [ ], { }, working in the innermost symbols first.
- 2. Simplify all exponential expressions.
- **3.** Perform all multiplication and division, working from left to right.
- 4. Perform all addition and subtraction, working from left to right.

### Student Notes

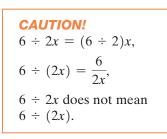
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Step (3) states that when division precedes multiplication, the division is performed first. Thus,  $20 \div 5 \cdot 2$  represents  $4 \cdot 2$ , or 8. Similarly, 9 - 3 + 1 represents 6 + 1, or 7.

## **EXAMPLE 3** Evaluate $5 + 2(a - 1)^2$ for a = 4. SOLUTION

 $5 + 2(a - 1)^{2} = 5 + 2(4 - 1)^{2}$ Substituting  $= 5 + 2(3)^{2}$ Working within parentheses first = 5 + 2(9)Simplifying 3<sup>2</sup> = 5 + 18Multiplying = 23Adding

3. Evaluate  $2(x + 1)^2 - 10$  for x = 5.



**4.** Evaluate  $8a^2 \div 5b - 4 + a$  for a = 5 and b = 2.

## Scheck Your

Choose from the following expressions an appropriate algebraic translation of each phrase.

- a) 0.06 x + 1
- **b)** x + y 6
- c) 3(x + y)
- **d)** 2(x y)
- **e)**  $\frac{1}{3}x$

**f**) 
$$\frac{x}{y} - 3$$

- 1. One-third of a number
- **2.** Six less than the sum of two numbers
- **3.** Twice the difference of two numbers
- **4.** One more than six percent of a number
- **5.** Three less than the quotient of two numbers
- **6.** The product of three and the sum of two numbers

### YOUR TURN

Step (3) in the rules for order of operations tells us to divide before we multiply when division appears first, reading left to right. This means that an expression like  $6 \div 2x$  means  $(6 \div 2)x$ .

**EXAMPLE 4** Evaluate  $9 - x^3 + 6 \div 2y^2$  for x = 2 and y = 5. SOLUTION

#### $9 - x^{3} + 6 \div 2y^{2} = 9 - 2^{3} + 6 \div 2(5)^{2}$ Substituting $= 9 - 8 + 6 \div 2 \cdot 25$ Simplifying 2<sup>3</sup> and 5<sup>2</sup> $= 9 - 8 + 3 \cdot 25$ Dividing = 9 - 8 + 75Multiplying = 1 + 75Subtracting = 76Adding

YOUR TURN

### **C. Sets of Numbers**

When evaluating algebraic expressions, and in problem solving in general, we often must examine the *type* of numbers used. For example, if a formula is used to determine an optimal class size, fractions must be rounded up or down, since it is impossible to have a fraction part of a student. Three frequently used sets of numbers are listed below.

### NATURAL NUMBERS, WHOLE NUMBERS, AND INTEGERS

### Natural Numbers (or Counting Numbers)

Those numbers used for counting:  $\{1, 2, 3, \dots\}$ 

### Whole Numbers

The set of natural numbers with 0 included:  $\{0, 1, 2, 3, \dots\}$ 

### Integers

The set of all whole numbers and their opposites:

 $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ 

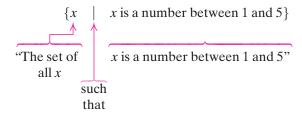
The dots are called ellipses and indicate that the pattern continues without end.

Integers correspond to the points on the number line as follows:



The set containing the numbers -2, 1, and 3 can be written  $\{-2, 1, 3\}$ . This set is written using **roster notation**, in which all members of a set are listed. Roster notation was used for the three sets listed above. A second type of set notation,

**set-builder notation**, specifies conditions under which a number is in the set. The following example of set-builder notation is read as shown:



Set-builder notation is generally used when it is difficult to list a set using roster notation.

**EXAMPLE 5** Using both roster notation and set-builder notation, represent the set consisting of the first 15 even natural numbers.

### SOLUTION

Using roster notation: {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30}

Using set-builder notation:  $\{n \mid n \text{ is an even number between 1 and 31}\}$ 

Note that other descriptions of the set are possible. For example,  $\{2x | x \text{ is an integer and } 1 \le x \le 15\}$  is a common way of writing this set.

YOUR TURN

The symbol  $\in$  is used to indicate that an **element** or a **member** belongs to a set. Thus if  $A = \{2, 4, 6, 8\}$ , we can write  $4 \in A$  to indicate that 4 *is an element* of A. We can also write  $5 \notin A$  to indicate that 5 *is not an element of A*.

**EXAMPLE 6** Classify the statement  $8 \in \{x \mid x \text{ is an integer}\}$  as either true or false.

**SOLUTION** Since 8 *is* an integer, the statement is true. In other words, since 8 is an integer, it belongs to the set of all integers.

YOUR TURN

Using set-builder notation, we can describe the set of all rational numbers.

### **RATIONAL NUMBERS**

Numbers that can be expressed as an integer divided by a nonzero integer are called *rational numbers*:

 $\left\{\frac{p}{q} \mid p \text{ is an integer, } q \text{ is an integer, and } q \neq 0\right\}.$ 

Rational numbers can be written using fraction notation or decimal notation. *Fraction notation* uses symbolism like the following:

 $\frac{5}{8}, \quad \frac{12}{-7}, \quad \frac{-17}{15}, \quad -\frac{9}{7}, \quad \frac{39}{1}, \quad \frac{0}{6}.$ 

In *decimal notation*, rational numbers either *terminate* (end) or *repeat* a block of digits.

For example, decimal notation for  $\frac{5}{8}$  terminates, since  $\frac{5}{8}$  means  $5 \div 8$ , and long division shows that  $\frac{5}{8} = 0.625$ , a decimal that ends, or terminates.

On the other hand, decimal notation for  $\frac{6}{11}$  repeats, since  $6 \div 11 = 0.5454 \ldots$ , a repeating decimal. Repeating decimal notation can be abbreviated by writing a bar over the repeating part—in this case,  $0.\overline{54}$ .

5. Using both roster notation and set-builder notation, represent the set of all multiples of 5 between 1 and 21.

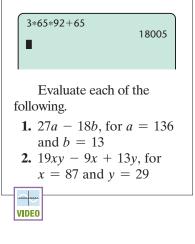
6. Classify the statement  $\frac{1}{2} \in \{x | x \text{ is a whole number}\}$ as either true or false.

### **Technology Connection**

Technology Connections are activities that make use of features that are common to most graphing calculators. Students may consult a user's manual for exact keystrokes. Most graphing calculators share the following characteristics.

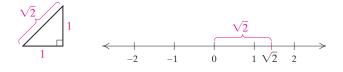
Screen. The large screen can show graphs and tables as well as the expressions entered. Computations are performed in the **home screen**. On many calculators, the home screen is accessed by pressing **ND (aur)**. The **cursor** shows location on the screen, and the **contrast** (set by **(ND) (a)** or **(ND) (c)**) determines how dark the characters appear.

**Keypad.** To access options written above the keys, we press **2ND** or **ALPHA** and then the key. Expressions are generally entered as they would appear in print. For example, to evaluate 3xy + xfor x = 65 and y = 92, we press  $3 \times 65 \times 92$  + 65 and then **ENTER**. The value of the expression, 18005, will appear at the right of the screen.



Many numbers, like  $\pi$ ,  $\sqrt{2}$ , and  $-\sqrt{15}$ , are not rational numbers. For example,  $\sqrt{2}$  is the number for which  $\sqrt{2} \cdot \sqrt{2} = 2$ . A calculator's representation of  $\sqrt{2}$  as 1.414213562 is an approximation since  $(1.414213562)^2$  is not exactly 2.

To see that  $\sqrt{2}$  is a "real" point on the number line, we can show that when a right triangle has two legs of length 1, the remaining side has length  $\sqrt{2}$ . Thus we can "measure"  $\sqrt{2}$  units and locate  $\sqrt{2}$  on the number line.



Numbers like  $\pi$ ,  $\sqrt{2}$ , and  $-\sqrt{15}$  are said to be **irrational**. Decimal notation for irrational numbers neither terminates nor repeats.

The set of all rational numbers, combined with the set of all irrational numbers, gives us the set of all **real numbers**.

### **REAL NUMBERS**

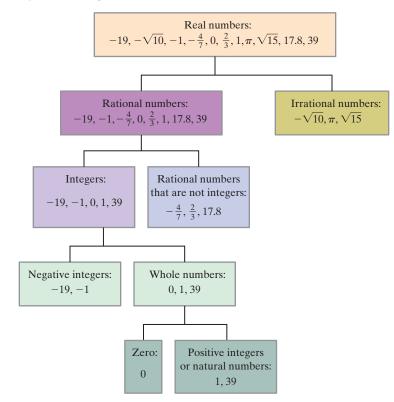
Numbers that are either rational or irrational are called *real numbers*:

 $\{x \mid x \text{ is rational or } x \text{ is irrational}\}.$ 

Every point on the number line represents some real number, and every real number is represented by some point on the number line.

$$\begin{array}{c} \text{Real} \\ \text{Numbers} \end{array} \left\{ \begin{array}{cccc} \text{Irrational numbers} & -\sqrt{2} & \sqrt{2} & \pi & \sqrt{15} \\ \text{Rational numbers} & -2 - \frac{5}{3} & -1 & -\frac{1}{2} & 0 & 1 & 1.4 & 2 & \frac{5}{2} & 3\frac{22}{7} & 4 \end{array} \right\}$$

The following figure shows the relationships among various kinds of numbers, along with examples of how real numbers can be sorted.



**EXAMPLE 7** Which numbers in the following list are (a) whole numbers? (b) integers? (c) rational numbers? (d) irrational numbers? (e) real numbers?

$$-29, -\frac{7}{4}, 0, 2, 3.9, \sqrt{42}, 78$$

### SOLUTION

- a) 0, 2, and 78 are whole numbers.
- **b)** -29, 0, 2, and 78 are integers.
- c)  $-29, -\frac{7}{4}, 0, 2, 3.9$ , and 78 are rational numbers.
- **d)**  $\sqrt{42}$  is an irrational number.
- e)  $-29, -\frac{7}{4}, 0, 2, 3.9, \sqrt{42}$ , and 78 are all real numbers.

### YOUR TURN

When every member of one set is a member of a second set, the first set is a **subset** of the second set. Thus if  $A = \{2, 4, 6\}$  and  $B = \{1, 2, 4, 5, 6\}$ , we write  $A \subseteq B$  to indicate that *A* is a subset of *B*. Similarly, if  $\mathbb{N}$  represents the set of all natural numbers and  $\mathbb{Z}$  is the set of all integers, we can write  $\mathbb{N} \subseteq \mathbb{Z}$ . Additional statements can be made using other sets in the diagram above.

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	Tutoring:
	Campus location
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	E-mail address
	Important supplements:
	(See the preface for a complete list of available supplements.)
	Supplements recommended by the instructor.
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## EXERCISE SET



Choose from the following list the word or words that best complete each statement.

- base constant division evaluating
- exponent irrational rational repeating
- terminating value variable

### FOR EXTRA MyMathLab<sup>®</sup>

- A letter that can be any one of a set of numbers is called a(n) \_\_\_\_\_.
- 2. A letter representing a specific number that never changes is called a(n) \_\_\_\_\_.
- 3. When x = 10, the \_\_\_\_\_\_ of the expression 4x is 40.

**7.** Which numbers in the following list are integers?

**Study Skills** *Get the Facts* 

Throughout this textbook, you will find a feature called *Study Skills*. These tips are intended to help improve your math study skills. On the first day of class, we recommend that you collect the course information shown here.

 $-245, 0, 15, \sqrt{11}, \frac{2}{3}$ 

- **4.** In *a<sup>b</sup>*, the letter *a* is called the \_\_\_\_\_\_. and the letter *b* is called the \_\_\_\_\_\_.
- 5. When all variables in a variable expression are replaced by numbers and a result is calculated, we say that we are \_\_\_\_\_\_ the expression.
- 6. To calculate  $4 + 12 \div 3 \cdot 2$ , the first operation that we perform is \_\_\_\_\_.
- 7. A number that can be written in the form a/b, where a and b are integers (with  $b \neq 0$ ), is said to be a(n) \_\_\_\_\_ number.
- A real number that cannot be written as a quotient of two integers is an example of a(n) \_\_\_\_\_\_ number.
- 9. Division can be used to show that  $\frac{7}{40}$  can be written as a(n) \_\_\_\_\_\_ decimal.
- **10.** Division can be used to show that  $\frac{13}{7}$  can be written as a(n) \_\_\_\_\_\_ decimal.

### A. Translating to Algebraic Expressions

Use mathematical symbols to translate each phrase.

- **11.** Five less than some number
- **12.** Ten more than some number
- **13.** Twice a number
- **14.** Eight times a number
- 15. Twenty-nine percent of some number
- 16. Thirteen percent of some number
- 17. Six less than half of a number
- 18. Three more than twice a number
- 19. Seven more than ten percent of some number
- **20.** Four less than six percent of some number
- **21.** One less than the product of two numbers
- 22. One more than the difference of two numbers
- 23. Ninety miles per every four gallons of gas
- 24. One hundred words per every sixty seconds

### **B. Evaluating Algebraic Expressions**

In Exercises 25–28, find the area of a square flower garden with the given length of a side. Use  $A = s^2$ . **25.** Side = 6 ft **26.** Side = 12 ft **27.** Side = 0.5 m **28.** Side = 2.5 m In Exercises 29–32, find the area of a triangular fireplace with the given base and height. Use  $A = \frac{1}{2}bh$ .



- **29.** Base = 5 ft, height = 7 ft
- **30.** Base = 2.9 m, height = 2.1 m
- **31.** Base = 7 ft, height = 3.2 ft
- **32.** Base = 3.6 ft, height = 4 ft

**To the student and the instructor:** Throughout this text, selected exercises are marked with the icon Ana!. Students who pause to inspect an Aha! exercise should find the answer more readily than those who proceed mechanically. This may involve looking at an earlier exercise or example, or performing calculations in a more efficient manner. Some Aha! exercises are left unmarked to encourage students to always pause before working a problem.

Evaluate each expression using the values provided.

**33.** 3(x - 7) + 2, for x = 10**34.** 5 + (2x - 3), for x = 8**35.**  $12 + 3(n + 2)^2$ , for n = 1**36.**  $(n-10)^2 - 8$ , for n = 15**37.** 4x + y, for x = 2 and y = 3**38.** 8a - b, for a = 5 and b = 7**39.**  $20 + r^2 - s$ , for r = 5 and s = 10**40.**  $m^3 + 7 - n$ , for m = 2 and n = 8**41.**  $2c \div 3b$ , for b = 2 and c = 6**42.**  $3z \div 2y$ , for y = 1 and z = 6**Aha!** 43.  $3n^2p - 3pn^2$ , for n = 5 and p = 9**44.**  $2a^{3}b - 2b^{2}$ , for a = 3 and b = 7**45.**  $5x \div (2 + x - y)$ , for x = 6 and y = 2**46.**  $3(m + 2n) \div m$ , for m = 7 and n = 0**47.**  $[10 - (a - b)]^2$ , for a = 7 and b = 2**48.**  $[17 - (x + y)]^2$ , for x = 4 and y = 1